ENGINEERING ANALYSIS OF THOROUGHBRED RACING

by

Rubin Boxer

Revelation Software

P.O. Box 21344
Santa Barbara, CA 93121-1344
Phone: (805) 962-9045
www.revelationprofits.com

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ABSTRACT:

While the process of assessing the capability of horses is, to a large extent, an art, some aspects of how a horse races can be analyzed.

We show that by considering the energy used by a horse, it is possible to derive "Capability Constants" which are inherent descriptors of the racing horse. In a further derivation we obtain the horse's internal "friction".

We develop a way to account for changes to the weight carried by a horse,

In essence, this paper provides an engineering dimension to describe and compare thoroughbred horses.

I-INTRODUCTION

The art of analyzing how a horse races is a fairly involved process. Some of the factors we have to consider are:

The horse's class which measures the quality of horses in its races.

The horse's breeding which determines whether a horse is better suited for long races (routes), short races (sprints), dirt or turf surfaces, and whether the horse can be expected to run well on a muddy track.

The horse's sex and age. Generally, male horses will defeat females and horses reach a peak and then decline with age.

The horse's physical fitness. How long has it been since its last race; has it been working out regularly; has it been improving?

The trainer's competence.

The jockey's competence.

The weight carried by the horse, or more important, the change in this weight since a previous race.

The horse's post position. Post positions are numbered from the track rail outward. A particular position can cause a horse, among other things, to run wide around turns adding to its race distance, or, to be blocked by other horses.

The type of race the horse runs, e.g., does the horse set the pace by being a front runner, or does the horse normally come from behind to vie for the lead?

The condition of the track. Track maintenance is done daily and between races. Each race itself affects the track. Wind, which, for convenience, we also associate with a track, can change the outcome of a race. Track related factors such as these cause racing performance to vary from day to day and race to race.

The differences among tracks. Some tracks are inherently faster or slower than others.
The horse's performance in previous races.

It may be presumptuous to apply the term "Engineering Analysis" to something as flesh-and-blood and non-mechanical as a thoroughbred race horse. The analysis, however, leads to

valid results and provides terminology that allows us to think about, and describe, horse racing in useful ways.

II-ANALYSIS

A. Capability Constants and Race Time.

Of all the factors noted above, we can argue that the most important indicator of a horse's potential capability is its performance in previous races.

We assert that a horse has a certain store of energy at the start of a race, which is used as the race progresses.

A simple analogy is useful. When a weight is dragged along the ground, energy is used in overcoming the friction between the weight and the ground. The greater the friction, the greater the energy use.

The case of a racing horse is, of course, much more complicated. The "friction" is caused by track condition effects, the contact between the horse's hooves and ground, internal interactions of muscle and bone, and other biological processes. We assume here that all factors can be included in what we term "equivalent friction".

We also assume that the rate at which horses can draw on this energy reserve varies. There is a maximum rate, which translates into maximum speed, and lower rates that yield lesser speeds. Whatever the rate of energy use, it is not sustainable indefinitely.

Horses can differ in their rates of energy expenditure. Consider sprint and route races. In sprints the horses usually race at full effort during the entire race. Speed near the start is at its maximum and then decreases. In routes there is often a need to save energy at the start for a greater effort at the end. In some instances, especially in turf races, horses will increase speed during part of the race.

The concept of "equivalent friction" is defined by equation 1:

$$f_e = uW \quad \text{(eq. 1)}$$

where:

f_e is equivalent friction force
u is friction coefficient
W is weight that the horse carries

We structure the analysis in terms of the more usual case where speed decreases, by assuming that the decrease is proportional to the energy used to overcome friction. (For convenience we consider constants of proportionality and units to be included in u.)

Let the distance along the track from the race start be x. Then:

$$v(t) = V_m - uWx(t) \quad \text{(eq. 2)}$$

where:

$v(t)$ is velocity, (speed)
V_m is starting velocity;
$x(t)$ is distance
t is time

With $v(t) = dx/dt$, equation 2 is a first order differential equation:

$$dx/dt = V_m - uWx(t) \quad \text{(eq. 3)}$$

Solving equation 3:

$$x(t) = (V_m/uW)(1-e^{-uWt}) \quad \text{(eq. 4)}$$

which gives the position of the horse at any specified time.

The horse's speed is:

$$v(t) = V_m e^{-uWt} \quad \text{(eq. 5)}$$

To obtain useful results we need numerical values for the constants in equations 4 and 5. If we let $k=uW$ for convenience, equation 4 becomes:

$$x(t) = (V_m/k)(1-e^{-kt}) \quad \text{(eq. 6)}$$

Equation 6 contains an exponential term which we approximate, using a series expansion, to make the calculation of the constants easier. This leads to:

$$x(t)=(V_m/k)(1-(1-kt+k^2t^2/2+..)) \quad \text{(eq. 7)}$$

Using the first terms of the series:

$$x(t) = V_m t - V_m k t^2/2 \quad \text{(eq. 8)}$$

Equation 8 has two unknowns, V_m and k. We can solve for them by using data from a previous race. We need the position of the horse, $x(t)$ at two times. One approach is to use the data at the first or second calls and at the end of the race. (The first and second calls are the times, from the start of the race, that the leading horse arrives at standard intermediate distances of the race.) The data, provided by a number of publications, also show how far the horse we are considering was behind the leading horse at the time of those calls. This gives us both the time and position of the horse.

The distance behind the leading horse is given in "lengths". Supposedly, this originated from the length of a horse itself. The data for lengths behind are subjective, provided by humans who observe the race. Values of about 10 or 11 feet have typically been used for a length; in this paper we use 10.

Using lengths behind as an actual distance is more accurate than equating a length to 1/5th second of time, as is often done, because the time to run one length varies at different points in the race.

In equation 8, let $z = V_m k$, then:

$$t_1^2 z - 2t_1 V_m = -2x_1 \quad \text{(eq. 9)}$$

where x_1 and t_1 are the position and time of the horse at the first call. A second equation would be similar for either the second call or finish of the race.

However, greater accuracy may be obtained, if we used more data, for example, three points. These can be the first and second calls and the race finish. Using finish time and position data an equation similar to equation 9, with variables of time and distance of t_f and x_f would be a third equation.

This is a case where the number of data points and equations that we have, 3, is larger than the number of unknowns, 2. A way to deal with this is to obtain a least mean squares fit. Only the results of that standard procedure are given here.

We define:

$$A = t_1^2+t_2^2+t_f^2$$
$$B = t_1^3+t_2^3+t_f^3$$
$$C = t_1^4+t_2^4+t_f^4$$
$$D = t_1x_1+t_2x_2+t_fx_f$$
$$E = t_1^2x_1+t_2^2x_2+t_f^2x_f \quad \text{(eq. 10)}$$

The results are:

$$V_m = (BE-CD)/(B^2-AC)$$
$$z = (2AE-2BD)/(B^2-AC) \quad \text{(eq. 11)}$$

With V_m, and using $k = z/V_m$ we have all the unknowns needed. We call V_m and k the horse's capability constants because they are useful in describing how the horse raced. More discussion of this is in Section III.
Having the capability constants, we can obtain the horse's time at any distance in the race from the quadratic form of equation 8.

$$(V_m/2)(kt^2)-V_m+x = 0 \quad \text{(eq. 12)}$$

The solution of this quadratic is:

$$t = 1/k - (1/V_mk)(V_m^2-2V_mkx)^{1/2} \quad \text{(eq. 13)}$$

In particular, if we enter x_f, the length of a race, we obtain t_f, the horse's finish time.

Next we derive a way to adjust for any change in weight that the horse carries.

B-Weight Change Effects

In the introduction, we noted that the performance of a horse will be affected by the weight it carries. In the United States, with the exception of a few tracks, as opposed to, say, Japan, the weight of the horse itself is not usually known. What we are given is the weight that the horse must carry.

Each race has certain weight conditions that must be met by each horse entered in the race. One number is provided which represents the weight of the jockey plus any weights added to meet those specified conditions. We are limited in our analysis to using the weight carried, without knowledge of the weight of the horse itself. This may not be too much of a limitation, since the actual weight of a fit horse remains fairly constant and a horse's baseline capability might be associated with that weight. The fact that our analysis yields results that agree with "rules of thumb" of trainers and other experts lends some credence to this conjecture.

From equation 4 we can obtain an expression for t, (without resorting to the series approximation that was used to simplify obtaining the capability constants):

$$t = \ln(1-kx/V_m)/-k \quad \text{(eq.14)}$$

Differentiating to obtain dt/dk:

$$dt/dk=(x/k)/(V_m-kx)+(1/k^2)(\ln((V_m-kx)/(V_m)) \quad \text{(eq. 15)}$$

We can obtain values for dt/dk, for different race lengths, x_f, using equation 15. To do this we need values for V_m and k. Although these vary, we can choose typical values by assuming that the weight results are not strongly sensitive to such variance. We choose

$$V_m = 0.0934$$
$$k = 0.0025$$

where the units of the basic equation terms are furlongs for distance and furlongs per second for velocity or speed.

Using $x = 8$, for an 8 furlong race, equation 15 yields:

$$dt/dk = 5042 \quad \text{(eq. 16)}$$

In horse racing, it is standard to deal in time units of 0.2 seconds, (1/5th of a second).

Using 0.2 seconds for dt in equation 16 yields 3.97×10^{-5} for dk which is 1.59% of the typical 0.0025 value for k. Since $k = uW$, and u the coefficient of friction is assumed constant, then whatever percentage change we obtain for k must be caused by the same percentage change in weight carried.

A typical value for weight carried is 116 pounds. A 1.59% change in this weight is 1.84 pounds. So for 8 furlong (1 mile) races, every change in weight of 1.84 pounds will change the horse's race time by 0.2 seconds.
The one fifth of a second standard time interval is called a "tick". It's useful to give the time change in terms of the number, or fraction, of ticks for each pound of weight change. Expressed this way, the above result is that for each pound of weight change in an 8 furlong race, the time a horse will take to run the race will change by 0.54 ticks.

In Table 1, we show the same result, for a number of common race distances.

TABLE 1
Time Change per One Pound Weight Change
Normalized to 116 pounds)

Race Distance (Furlongs)	Time Change (Ticks)
5	0.187
5.5	0.231
6	0.280
6.5	0.336
7	0.398
7.5	0.467
8	0.543
8.5	0.627
9	0.719
10	0.930
12	1.479
15	2.716
16	3.276

(Ainsley [Reference 1.] refers to a weight formula used by racing experts: At sprint distances, (say 6 furlongs), four pounds slows a horse by 1/5th second. Three pounds does it at a mile, (8 furlongs), two pounds for a mile and one eighth, (9 furlongs), and one pound for a mile and a quarter, (10 furlongs). Using the pound per tick format of Table I, Ainsley's numbers are equivalent to 0.25, 0.33, 0.5, and 1.0 pounds at these distances. Agreement is close to our results at 6 and 10 furlongs, and "ballpark" at the other distances.)
Our example uses 116 pounds as a typical weight. In the table we retain this weight as a standard because the basic calculation involves a percentage change from a particular weight. Other weights would lead to different values. In practice this is not a problem, as we explain in part of the discussion that follows.

III-DISCUSSION

A. Capability Constants Discussion.

We derived V_m and k and called them the capability constants of the horse. The interpretation of V_m is straight forward; it's the speed of the horse just after the start of the race. In some races this turns out to be important because it yields some idea of which horses "set the pace", i.e., are out in front at the start.

What interpretation can we put on k? Recall that we defined $k = uW$. This is also the equivalent friction of equation 1. From equations 4 and 5, we see that k can also be thought of as a degradation factor since it appears as a negative power of the exponential terms in those equations.

The larger the value of k, the faster the horse uses energy, and the faster is its drop in speed. If k were to equal zero it would mean that the horse ran at constant speed, while a negative value of k would result from a horse that sped up.

We must exercise care when interpreting k. Equivalent friction was described as being made up of components which depend on the surface condition of the track and the physical condition of the horse. If the track condition is such that it is termed "slow", meaning that it causes horses to tire easily, k will be large. Without regard to track condition however, if a horse is not in fit condition when it races, it can tire easily, also causing a large value for k.

To evaluate k, for the horse itself, we need to separate the part of race performance due to track condition from the part due to horse capability. Since both are contained in equivalent friction, the equations alone do not allow us to separate the effects.

One way to deal with this is to use the concept of track variant.

The need to separate horse capability from track condition effects is not a problem unique to the engineering analysis given here. It is classical to handicapping. As a result, there are many sources of information available which describe the condition of tracks for particular days and races. Numbers are provided to adjust race times for track condition. Such numbers are called track variants.

Track variants are derived by experts who account for race class, and the way races are run. They describe track condition in terms of these variants. If we use these variants at the calls and finish of a race, to adjust the times at those points, then k will be based more on horse capability than on track condition.

B. Race Time Discussion.

We showed that we can take data from a previous race to derive the capability constants which can be used in equation 13 to obtain race time. Of course we already know the race time of a previous race. But we have accomplished two things. First, we have a method that analyzes races and gives us the constants that describe how particular horses raced. Second, we have tools that can be used to try to predict the outcome of future races.

Factors that make prediction difficult were touched on in Section I. Here we show how to deal with some of them.

To compare horses we need to proceed with a sequence of steps. These steps have both judgmental and analytical aspects to them, i.e., some art and some science. Typically we start by selecting a previous race.

Let's say we are going to examine today's races. Publications like the, "Daily Racing Form", provide us with data about each horse's previous races. (Some races have horses that never raced before, so no race data is available for them. For such horses we obviously cannot apply our methods but must rely on clues from a horse's breeding and it's workouts.)

Since previous race data is fundamental to our approach, selecting that race is all important. Some things to consider:

It's preferable to use races that were at today's distance, surface, and class. If a race at today's distance is not available choose one close to it.

Choose a recent race over an earlier one as an indicator of a horse's present fitness. (Even this simple statement has a caveat. If a horse hasn't raced in a "reasonable" time, then it might be better to choose the earlier race because the horse may have needed the layoff and rest period to return it to an earlier fitness. Recent good workouts may be a sign that this is so.)

After choosing a previous race the next step is to use its data to obtain the capability constants and an estimate of finish time. (The description of how to do this, which follows, may seem tedious, but bear in mind that it is easily accomplished, accurately, with a computer.

We adjust horse position at the calls and finish:

$$x_{1a} = x_1 - FL_1$$
$$x_{2a} = x_2 - FL_2$$
$$x_{fa} = x_f - FL_f \quad \text{(eq. 17)}$$

where,

x_{1a}, x_{2a}, and x_{2a} are the call and finish positions after adjustment.

L_1, L_2, and L_f are the lengths behind at the calls and finish.

$F = 1/66$, furlongs per length.

Earlier we showed how track variant and weight carried affect race data; so, we must take them into account.

A useful way is to "normalize" previous races. We adjust the race results so that they represent horses carrying 116 pounds racing on a normal track. A normal track is one where the horses race at "par", i.e., the race outcomes are what we expect from horses of their class and racing style. (One measure of a par track is a variant of zero in a method used by the publication, "Today's Racing Digest". In their system, variants are given in number of "ticks". Positive numbers mean that the track was fast so they are added to finish time, while negative numbers mean the track was slow and are subtracted.) If the previous race was at a different track, then we add a refining adjustment to account for their speed differences. Some sources provide variants for each of the calls. In our examples, we assume we have track variant for finish time only.

Since both weight carried, and track variant affect race time and are measured in "ticks", we can deal with them together to normalize call and finish times:

$$t_{1a} = t_1 + .2(x_{1a}/x_{fa})(p+(w_f)(116-W_p))$$
$$t_{2a} = t_2 + .2(x_{2a}/x_{fa})(p+(w_f)(116-W_p))$$
$$t_{fa} = t_f + .2(p+(w_f)(116-W_p) \qquad \text{(Eq. 18)}$$

where, t_{1a}, t_{2a}, and t_{fa} are adjusted call and finish times; p is track variant, w_f is the weight factor in Table I, and W_p is the weight carried in the previous race.

Equations 18 show that weight and variant time adjustments are distributed over the track in proportion to the call and finish distances. It's correct to expect that this is proper for weight. For example, at the half mile point of a mile race, the time change due to weight should be half that at the finish. For variant, however, this may not be the case. Track conditions can vary during the race, e.g. a headwind can change to a tailwind. But in the absence of variant numbers at the internal race calls, the assumption of a proportional distribution seems reasonable.

Equations 17 and 18 provide us with all we need to apply equations 10, 11, and 13 to obtain the capability constants and finish time. (We use the adjusted time and position values in equations 10, 11, and 13.) If we make these calculations for each horse in a race, we obtain finish times for all so we have a predicted order of finish. This prediction is based on normalized results. (Recall normalized results are for horses carrying 116 pounds racing on a par track.) In today's race, horses may be carrying different weights and the track will probably not be at par. We can argue that we need not concern ourselves with the value of "today's" track variant because, unless there are special conditions present, each horse, racing over the same track, will be affected in the same way. The predicted finishing order should not change.

We make the time adjustment due to weight:

$$.2(116-W_t)(w_f)$$

where W_t is the weight carried today. Adding this time adjustment to each horse leads to a "denormalized" finish order prediction for "today's" race.

Now and again, a few further refinements are possible. The data of previous races include qualitative comments such as:

"Four-wide into turn"
"Wide early"
"Broke slowly"
"Boxed in"

These comments can be very important. The first two are examples of where a horse may actually have run a longer race; the last two are examples of a horse that might have been able to run its race in a faster time.

If an entire race were run without turns, a horse that ran wide would not be at a disadvantage. But almost every race has at least one turn, so horses that run wide around a turn have to cover more ground, and the extra distance can be appreciable.

To see this consider the distance around one turn, i.e., half a circle:

$$C = \pi r \quad \text{(eq. 19)}$$

where C is the distance around the turn, and
r is the radius of the circle that the rail horse travels around the turn. (π, of course, is the ratio of a circle's circumference to its diameter, approximately equal to 3.1416.)

If the separation of horses racing side by side is s, then the distance traveled by those racing wide around the turn is:

$$C_n = \pi(r+ns) \quad \text{(eq. 20)}$$

where n takes on the values, 1, 2, 3, etc., representing wide horse positions starting with the horse next to the rail horse.

The extra distance, α, around a turn traveled by a non-rail horse, compared to the rail horse, is the difference between C_n and C:

$$\alpha = \pi ns \quad \text{(eq. 21)}$$

This equation of extra distance does not depend on circle radius which means that it is correct regardless of the length of the turn.

If a conservative value for horse separation, s, is 3 feet then the extra lengths around a turn is shown in Table II.

TABLE II

Extra Distance Around a Turn

Position from Rail Horse (Number)	Extra Distance (Lengths)
1	0.94
2	1.88
3	2.83
4	3.77
5	4.71

As we can see, the extra lengths a wide horse must travel can make a difference in race outcome. If the separation between horses is greater than the assumed 3 ft, as it can very well be, the extra distance traveled is greater still. Further, if a horse goes wide early and stays there throughout a two turn race, the extra distance traveled doubles.

C. Additional Discussion.

Other important facts about a race are available from our equations. For example, once the capability constants are obtained, equation 5 can be used to obtain a horse's speed at different times, which in turn means that the ratio of speeds at the calls and finish can be calculated. These internal call and finish values can be useful in evaluating a horse's form.

We showed that the normalizing process gave us information on how the horses would have raced on a par track. In other words the effects of track variant were eliminated. This means the capability constants refer more to the horse itself than to track condition. To the extent this is true, we can obtain an estimate of a horse's "equivalent friction", independent of the track.

Since $k = uW$:

$$u_{eh} = k/W \quad \text{(Eq. 22)}$$

where u_{eh} is the equivalent friction coefficient of the horse itself. A low value of u_{eh} means that the horse ran efficiently and implies that the horse was fit. A negative value may also imply that a horse was able to race without reaching its limit on rate of energy use. Further study of this conjecture is needed.

V-CONCLUSION

This paper develops tools to analyze thoroughbred race horses.

We should bear in mind that the non-mechanical nature of horses, and the occurrence of the unexpected during a thoroughbred horse race, caution us that no analytical method is infallible. In a statistical sense, however, we expect the better horses to win, and a method that helps us identify such horses is valuable. The engineering approach discussed in this paper does that.

Reference:

1. Ainsley's Encyclopedia of Thoroughbred Handicapping; Ainsley, Tom, William Morrow &

Company, New York, 1978; Page 253